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A Possible Gravitational Planck's Constant†

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Abstract

The hypothesis is made that Planck's constant involved in gravitational interactions, has a value different from that measured in electromagnetic interactions. An attempt is made to give a value to this gravitational constant.

Classical gravitation theory does not deal with Planck's constant. Attempts made until now to build a quantum theory of gravitation always assumed that Planck's constant has the same value both in electromagnetic and in gravitational phenomena. On the other hand, all that we know about Planck's constant comes out from 'electromagnetic' experiments (footnote 1) or from strong interactions. No purely gravitational measurement of Planck's constant exists.

It seems that the equality assumption was never justified and it is believed that it was made on a 'no reason to change' basis. At the same time there does not seem to exist any theoretical or experimental fact supporting the equality of the two values.

Lets examine an example in which the equality hypothesis does not look completely satisfactory. This is the solution of the problem of gravitational radiation from a rotating rod. The solution given by general relativity for a rod rotating at a practical speed shows that gravitational radiation is to be expected. The frequency of this radiation should be equal to the angular frequency of rotation. The amount of energy radiated in a second by a labora-

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[1] I.e. experiments in which the detecting system and the phenomenon itself depend on electromagnetic interaction.

tory sized rod should be of the order of 10^{-37} Joule/sec (footnote 2). If the energy is assumed to be radiated in quanta, each carrying the energy:

$$E = h\nu \tag{1}$$

the number given means that the rod should radiate a gravitational quantum (a graviton) each 10^6 seconds. Since the radiating objects are individual particles in the rod and there is no sign compensation as for charges, the conclusion has to be drawn that elementary particles emit gravitational radiation with an extremely small probability and with an unknown law. Many less believable explanations can be given.

Notice now that a very small value of h in equation (1) could give rise to the conclusion that each individual particle emits a graviton with a very small energy associated.

Lets try now to give a value for Planck's constant involved in gravitational phenomena. Consider an oscillator consisting of two oppositely charged electrons oscillating around their centre of mass. Such a system emits electromagnetic radiation with frequency equal to the oscillation frequency. Gravitational radiation with the same frequency is to be expected from the mass property of the electrons.

Energy carried out by electromagnetic quanta is proportional to the change of the electromagnetic potential energy of the electrons in one cycle of oscillation. Energy carried out by gravitational quanta has to be proportional to the change of the gravitational potential energy of the electrons if energy conservation is to be hold.

The ratio of the radiated energies is:

$$\frac{Egr}{Eelm} = G\frac{m^2}{q^2} \simeq 2.4 \cdot 10^{-43} \tag{2}$$

The simplest dependence of E on ν will be stated:

$$Egr = H\nu \tag{3}$$

However, it is to be stressed that the emitting system under consideration is more symmetrical with respect to masses than with respect to charges so that the frequency of the gravitational wave is expected to be twice the frequency of the electromagnetic one. Then

$$Egr = H(2\nu) \tag{4}$$

From equation (2)

$$\frac{H(2\nu)}{h\nu} = G\frac{m^2}{q^2} \simeq 2.4 \cdot 10^{-43} \tag{5}$$

[2]See for instance J. Weber (1964) 'Gravitation and Relativity', Chu and Hofmann editors, p. 90, Benjamin, New York.

and from this

$$H = \tfrac{1}{2} G \frac{m^2}{q^2} h \tag{6}$$

$$H \simeq 7.95 \cdot 10^{-77} \ J \cdot \text{sec} \tag{7}$$

The value just calculated for the gravitational Planck's constant depends on m^2. This result is apparently paradoxical, since we are accustomed to the one-valued electromagnetic Planck's constant. This takes up the same value for any emitting system due to the fact that charge takes only one value. Equation (6) predicts that gravitational waves of the same frequency can carry different amounts of energy owing to the difference in the sources of the waves.

If indetermination relations have to be taken to hold in purely gravitational phenomena, the Planck's constant to be used is the gravitational one

$$\Delta E \, \Delta t \simeq \frac{H}{2\pi} \tag{8}$$

so that gravitational probes should offer a much finer resolution than electromagnetic ones do. As a matter of fact a gravitational wave of energy E should have a wavelength some 40 orders of magnitude smaller than that of an electromagnetic wave of the same energy.

If we come back to the example of the rotating rod, we can use the gravitational Planck's constant relative to nucleons to calculate the rate of radiation. The number of gravitons emitted per second is found to be in the same order of magnitude as the number of nucleons in the bar.

Two more examples will be given of equations in which the calculated value of Planck's constant seems to work better than the electromagnetic one. The first regards the characteristic length. This is given by $L = (hc)^{1/2} = 1.78 \cdot 10^{-13}$ m for the electromagnetic field. The characteristic length for the gravitational field is given by: $L^* = (hG/c^3)^{1/2}$. This yields a value $L^* \simeq 10^{-33}$ m if h is used. This value is very far apart from any other physical length. If H is inserted, L^* takes up the value $L^* = 1.77 \cdot 10^{-56}$ m which is within one order of magnitude of the Schwarzschild radius of the electron. In a similar way, the Compton wavelength $\lambda = h/mc = 3.87 \cdot 10^{-13}$ m takes up a value $\lambda^* = 4.63 \cdot 10^{-56}$ m if H is used instead of h.